Energy Production in the Sun
(What makes the Sun Shine?)

Relativity and Astrophysics
Lecture 29
Terry Herter

Outline

- Energy production in the Sun
  - Stellar reactions (H => He): P-P chain & CNO cycle
  - Energy release / budget
  - Transport of energy

- Reading
  - Spacetime Physics: Chapters 8

- Homework (due Wed. 11/11/09)
  - 8-9, 8-14, 8-41, 8-6
Some Energy Examples

- 1 erg = A snowflake hitting the ground.
- 1000 J = Energy a match produces
- $8.4 \times 10^6$ J = 2000 Calories = Energy the body uses in a day
- 330,000 J = Boils 1 quart of water = Runs a 60 W light bulb for 1.5 hrs
- 400,000 J = Energy of a 1 ton car at 60 mph
- $4 \times 10^9$ J = 1 ton TNT
- $10^{15}$ J = Nuclear explosive (250 kilotons TNT)
- $10^{25}$ J = Solar Flare

What makes the Sun shine?

- The Sun emits $4 \times 10^{26}$ Watts
  - People power about 600 billion Watts
  - US Power consumption about $10^{13}$ Watts
  - Equivalent to $>$100 billion nuclear bombs/sec!
- We know from dating of rocks on the Earth and Moon, that the Sun is at least 4.5 billion years old.
  - $\Rightarrow$ Need about $6 \times 10^{43}$ J of energy
Possible energy sources stars*

- **Chemical Reactions**
  - Such as a burning fire
  - Sun’s lifetime would be ~1000 years!!

- **Gravitational Compression**
  - Shrinking – use gravity, like a water fall
  - Must collapse ~50 feet per year!
  - Sun’s lifetime would be ~15×10^6 years.

- **Nuclear reactions**
  - Convert mass to energy
  - Much more energy per unit mass than chemical reactions

*Main sequence stars

Stellar Cores

- \( T_{\text{core}} \sim 15 \times 10^6 \) K
- Particles move very fast.
  - Protons can overcome “coulomb barrier” and collide with other nuclei.

- Collision results in:
  - Deuterium + Positron + Neutrino + Energy
  
\[ ^2_1H + e^+ + \nu_e \]
The Neutrino (Nature’s Ghost)

- A particle produced in stellar nuclear reaction is the neutrino, designed by the symbol $\nu$.
- $\nu$: has no charge & small mass (close to zero)
  - very little interaction with matter
- The Sun is transparent to neutrinos.
- “Neutrino telescopes” can look at the interior of the Sun.
  - More than $5 \times 10^{13}$ solar electron neutrinos pass through the human body every second. (~$6.5 \times 10^{10}$ solar neutrinos pass through every square centimeter on Earth)
- Original experiment, tank of chlorine (cleaning fluid) in South Dakota mine.
  - $\nu$’s occasionally interact to convert $^{37}\text{Cl}$ to $^{37}\text{Ar}$
  - There appeared to be too few $\nu$’s!
  - This problem was solved by the finding that neutrinos oscillate between three flavors (electron, muon, and tauon) as they pass through matter and space
- In 2002 Nobel prize won by Ray Davis Jr. and Masatoshi Koshiba for studies of solar/cosmic and first real time detection of supernova neutrinos, respectively

Proton-Proton Chain

- Converts H to He
- Most efficient in lower mass stars like the Sun
  - $T_{\text{core}} > 10,000,000 \text{ K}$
Energy Production in the Sun

CNO Cycle
- $^{12}\text{C}$ is a catalyst for $\text{H} \rightarrow \text{He}$ reaction
- Most efficient in higher mass stars
  - $T > 16,000,000 \text{ K}$
- Hans Bethe (Cornell) 1939

\[
\begin{align*}
^{12}\text{C} & \rightarrow ^{13}\text{N} \\
^{13}\text{N} & \rightarrow ^{14}\text{N} \\
^{14}\text{N} & \rightarrow ^{15}\text{O} \\
^{15}\text{O} & \rightarrow ^{12}\text{C} + \gamma \\
\end{align*}
\]

Energy release from H $\rightarrow$ He
- Energy is produced in the Sun by converting H to He.
- The energy released is mainly in the form of photons
  - The mass difference between H & He gives the energy released

\[
\begin{align*}
4 \text{H}^1 & = 6.69048 \times 10^{-27} \text{ kg} \\
- 1 \text{He}^4 & = 6.64648 \times 10^{-27} \text{ kg} \\
\Delta(\text{mass}) & = 0.04400 \times 10^{-27} \text{ kg} \\
\end{align*}
\]

- $E = mc^2 \Rightarrow 4.0 \times 10^{-12} \text{ Joules per reaction}$

- If 10% of the hydrogen is converted into helium for the sun, this would produce about $10^{44} \text{ J}$ of energy.
  - Enough to keep the Sun burning for about 10 billion years
Energy Production in the Sun - 1

- The solar constant (luminous energy from the Sun at the Earth) is 1372 W/m². We have
  - \( R_{\text{earth}} = 6.4 \times 10^6 \text{ m}, d_{\text{earth-sun}} = 1.5 \times 10^{11} \text{ m}, M_{\text{sun}} = 2.0 \times 10^{30} \text{ kg} \)

- How much mass is converted to energy every second in the Sun to supply energy to the Earth?
  - We have 1 W = 1 Joule/sec = 1 kg/m/s². To get the solar constant in units of mass (kg), we have
    \[
    \frac{1372 \text{ J/s}}{c^2} = \frac{1.372 \times 10^3 \text{ kg m}^2/\text{s}^3}{9.00 \times 10^9 \text{ m}^2/\text{s}^2} = 1.524 \times 10^{-14} \text{ kg/s}
    \]
  - The total energy falling on the Earth is the solar constant times the projected area of the Earth (as seen from the Sun), so
    \[
    A_{\text{sunSC}} = (1.3 \times 10^{14} \text{ m}^2) \times (1.5 \times 10^{-14} \text{ kg/m}^2) = 2.0 \text{ kg/s}
    \]
  - which give the mass converted every second to supply light incident on the Earth (not the total luminosity of the Sun)

Energy Production in the Sun - 2

- The solar constant (luminous energy from the Sun at the Earth) is 1372 W/m². We have
  - \( R_{\text{earth}} = 6.4 \times 10^6 \text{ m}, d_{\text{earth-sun}} = 1.5 \times 10^{11} \text{ m}, M_{\text{sun}} = 2.0 \times 10^{30} \text{ kg} \)

- What is the total mass converted to energy every second in the Sun to supply luminous energy?
  - The solar constant is the same to every part of a sphere surrounding the Sun at the distance of the Earth, so still have
    - solar-constant rate = 1.524 \times 10^{-14} \text{ kg/s}
  - Multiplying by the area of the sphere we get mass converted every second to support the total luminosity of the Sun
    \[
    4\pi d_{\text{earth-sun}}^2 \times SC = (2.8 \times 10^{23} \text{ m}^2) \times (1.5 \times 10^{-14} \text{ kg/s}) = 4.2 \times 10^9 \text{ kg/s}
    \]
  - A metric ton is 1000 kg, so about 4 million metric tons per second are converted to energy.
Energy Production in the Sun - 3

- The solar constant (luminous energy from the Sun at the Earth) is 1372 W/m². We have
  - \( R_{\text{earth}} = 6.4 \times 10^6 \text{ m}, d_{\text{earth-sun}} = 1.5 \times 10^{11} \text{ m}, M_{\text{sun}} = 2.0 \times 10^{30} \text{ kg} \)

- Most of the Sun energy is from burning H to produce He. How many metric tons of H are converted to He to supply this energy?
  - Take \( m_p = 1.67262 \times 10^{-27} \text{ kg} \) and \( m_{\text{He}} = 6.64648 \times 10^{-27} \text{ kg} \)
  - The mass difference between four protons (\( 6.69048 \times 10^{-27} \text{ kg} \)) and one helium nucleus (2 proton + two neutrons) is \( 0.04400 \times 10^{-27} \text{ kg} \)
  - Thus the ratio of H-burned to converted mass is \( 6.69048/0.04400 = 150 \) or about 0.7% of rest energy (mass) of the original H is converted into He. Hence the total mass burn rate of the Sun is
    \[
    \text{total mass burn rate} = 150 \times 4.2 \times 10^9 \text{ kg/s} = 6.3 \times 10^{11} \text{ kg/s} \quad \text{H} \rightarrow \text{He}
    \]
  - Or about 630 million metric tons each second

A2290-29 Energy Production in the Sun - 4

- The solar constant (luminous energy from the Sun at the Earth) is 1372 W/m². We have
  - \( R_{\text{earth}} = 6.4 \times 10^6 \text{ m}, d_{\text{earth-sun}} = 1.5 \times 10^{11} \text{ m}, M_{\text{sun}} = 2.0 \times 10^{30} \text{ kg} \)

- How long will the Sun continue to produce energy (ignoring other processes and emissions from the Sun)?
  - The mass of the Sun is determined by applying Kepler’s harmonic law \( (P^2 \propto a^3 / M_{\text{sun}}) \) which was proved by Newton.
  - Dividing the burn rate into the fuel supply (total mass) gives an estimate of how long the Sun will last
    \[
    \text{lifetime} = \frac{M_{\text{sun}}}{\text{burn rate}} = \frac{2.0 \times 10^{30} \text{ kg}}{6.3 \times 10^{11} \text{ kg/s}} = 3.2 \times 10^{18} \text{ s} = 10^{11} \text{ yr}
    \]
  - However, the Sun will convert only about 10% of its H mass to He, so this estimate is high by a factor of ten. The main-sequence lifetime of the Sun (the H -> He burning phase) is \( \sim 10^{10} \text{ yr} \).
Star stuff

- The conversion of H into He is not the only nuclear reaction than can take place in stars.
- All elements other than H and He are produced from stars (or explosions of stars.)
- The material in you was formed by a star!
  - Elements such as C, N, O, and up to Fe (iron) can be produce in stars (if the mass is high enough)
- Heavily elements are produce in supernova explosions
  - Lighter nuclei are bombarded with neutron during the explosion

Energy Transport in Stars

- Fusion reaction produce the energy in the central cores of stars
- How does the energy produced get out?
- Energy can be transported by
  - Conduction
  - Convection
  - Radiation
- Stars use the latter two methods
  - Trade between convection and radiation depends on the star and region within a star
A Model of the Sun

<table>
<thead>
<tr>
<th>Temp. (10^6 K)</th>
<th>Density g/cm^3</th>
<th>Energy Transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>~15</td>
<td>100</td>
</tr>
<tr>
<td>Rad.</td>
<td>~3</td>
<td>1</td>
</tr>
<tr>
<td>Conv.</td>
<td>~1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

T_{surface} \approx 6000 \text{ K}

The Interiors of Stars

Massive stars (> 2 M_{sun}) have small convective cores and large radiative envelopes.

Low mass stars (< 1 M_{sun}) have small radiative cores and large convective envelopes.