Spacetime Diagrams

Relativity and Astrophysics
Lecture 21
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Outline

- Fate of the Universe
  - Forever expansion
  - Dark Energy
- Time is Private
- Spacetime diagrams

Prelim
- Wednesday, Oct. 21
- Closed book and notes, will cover lectures 11 – 20
  - But some of this material depends on earlier lectures
- Should know
  - Will have both qualitative and quantitative questions
  - Most equations will be provided (if needed)
**The Two Omegas**

- So now we only need to know $\Omega_M$ and $\Omega_\Lambda$.
- With zero (no) cosmological constant
  - Have $\Omega_M$ and $\Omega_k$ with $\Omega_k = 1 - \Omega_M$
- With a non-zero cosmological constant
  - Have $\Omega_M$, $\Omega_\Lambda$, and $\Omega_k$ with $\Omega_k = 1 - \Omega_M - \Omega_\Lambda$

**Determine Dynamics and Geometry**
- $\Omega_M$ and $\Omega_\Lambda$ set acceleration/deceleration
- $\Omega_k$ sets geometry ($\Omega_k = 1 - \Omega_M - \Omega_\Lambda$)

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**$\Lambda \neq 0$**

Expansion velocity in the past depends upon $\Omega_M$ and $\Omega_\Lambda$. Galaxies would be moving faster in the past for a non-empty universe.
Cosmic Acceleration: Updated

- The results from 2004 have not appreciably changed.
- Here are some results from 2007.

Figure from: Riess, Turner, SciAm, Feb. 2004

Spacetime Diagrams

The Cosmic Triangle
(with the Three Omegas)

Matter +
Cosmological
Constant +
Curvature

Ω

Ω_M
1.0

Ω_K
0.0

Ω_Λ
1.0

OPEN

CLOSED

decelerating
accelerating

expands
forever

eventually
recollapses

Observations in
the Cosmic Triangle

- "CMB" refers to measurements of structure in the Cosmic Microwave Background
- "Clusters" refers to mass estimates of clusters of galaxies
- "SNe" refers to data from Type Ia Supernovae.

Intersection gives Omegas consist with all the data

See: http://www.sciencemag.org/cgi/content/full/284/5419/1481
Dark Energy

- The acceleration of the Universe driven by an enormous amount of energy.
- What causes the acceleration?
  - Or what is the cosmological constant?
  - Unknown!
- This unknown energy is called “dark energy”.
  - Attempts to estimate it from particle physics are off by at least a factor of $10^{55}$!

Even more of the Universe is unknown to us than we thought!

Illustration from WMAP website
The Fate of the Universe

- The best estimates now yield
  - $H_o = 71$ km/sec/Mpc
  - $\Omega_M = 0.27$, $\Omega_\Lambda = 0.73 \Rightarrow \Omega_k = 0$
  - $\Rightarrow$ Age of universe is 13.67 Gyr

- This implies
  - The Universe will go on expanding forever
  - Because $\Lambda > 0$, it will expand ever faster!

Time is a Private Matter

- Time is a private quantity
  - It is carried with the particle
  - Time is not a public quantity – that is, there is no absolute time

- Time is route dependent!
  - Time depends on the path a particle takes
  - This differs from a quantity such as temperature
Spacetime diagram (or map)

- Plots space on one axis and time on the other
- Payoffs
  - Place space and time on equal footing
  - Review history of events and motions along a given line in spacetime
  - Plot same events in different spacetime diagrams for different inertial reference frames
    - Can find what is different and what is the same between the frames

Reference event O, along with other events A, B, C, and D. The dashed horizontal line represents events simultaneous in time while the dashed vertical line represents events at the same location.

Same events, different frames

- Considers a light flash reflected off a mirror and back as seen from three different frames
  - rocket, lab, and super-rocket
  - The mirrors are at rest in the rocket frame and the super-rocket is traveling in the same direction but faster (relative to lab frame) than the rocket
- The light paths look like that shown below:
The spacetime interval is conserved (the same for each frame).

\[(\text{interval})^2 = (\text{time})^2 - (\text{space})^2 = \text{constant}\]

- The red hyperbola in the spacetime plots show the line of constant interval.
- The arrows represent the same arrow in spacetime!
- Maps show different perspectives of the same arrow in spacetime.
- Spacetime geometry is Lorentz geometry, not Euclidean.