Doppler Effect

Relativity and Astrophysics
Lecture 10
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Outline

- Doppler Shift
- The Expanding Universe – Hubble’s discovery

Reading
- Spacetime Physics: Chapter 4
- Problem L-2, page 112 (due today/Monday)
  - Will hand back on Monday if you hand it in on today

Prelim

- Wednesday, Sept. 23
- Closed book and notes, will cover material through today
- Should know:
  - Spacetime interval, fundamental postulates of Special Relativity, simultaneity and other issues raised by relativity
  - Will have both qualitative and quantitative questions
  - Lorentz Transformation equations will be provided (if needed)
Doppler Effect

The Doppler effect is extremely important for Astronomy.

- This is a shift in the frequency (wavelength) of light from an atom due to motion with respect to the observer.
- It is the Doppler effect the allows measurement of the expansion of the Universe (resulting from the Big Bang)

Following problem L-5 in the textbook we will derive the Doppler shift.

Doppler Shift

Consider a rocket which has source that emits pulse with a frequency $f'$ (pulses or "waves" per second).
- Suppose these waves travel in the positive $x$-direction with a speed $c$.
- Thus distance between pulses will be $c/f$ (velocity $\times$ time).

Setting up the origins
- Let the 0th pulse pass the origin ($x = 0$) at $t = 0$ and choose the origin of the rocket to pass the origin of the laboratory at the same time.

Lab Frame:
- The 0th pulse moves to the right with the speed of light, so its position at any given time $x = t$.
- The next pulse is delayed by a distance/time $c/f$, while the 2nd pulse is delayed by $2c/f$, etc. Thus the position for the nth pulse is given by

$$x = t - \frac{n}{f} \frac{c}{f} \Rightarrow \quad n = \frac{f}{c}(t-x)$$

For the lab frame.
Doppler Effect

**Doppler Shift**

- We can perform the same argument to get the equivalent relationship for the rocket frame.
  \[ x' = t' - n \frac{c}{f} \Rightarrow n = \frac{f'}{c} (t' - x') \]

- Now apply the inverse Lorentz transformation (moving from the rocket frame to the lab frame).
  \[ t' = \gamma t - v_{rel} \gamma x \quad x' = \gamma x - v_{rel} \gamma t \]

- To get
  \[ n = \frac{f'}{c} \left( (t - v_{rel}x) - (\gamma x + v_{rel} \gamma t) \right) = \frac{f'}{c} (1 - v_{rel}) (t - x) \]

- Substituting for \( \gamma \) gives
  \[ n = \frac{f'}{c} \left( \frac{1 + v_{rel}}{1 - v_{rel}} \right)^{1/2} (t - x) \]

**Doppler Shift**

- Set the two equations for \( n \) equal (since they much correspond to the same pulse (event) gives:
  \[ f = f' \left( \frac{1 + v_{rel}}{1 - v_{rel}} \right)^{1/2} \]

- For pulse going in the negative \( x \)-direction
  - We have the equivalent equations but the pulse is moving to the left.
    \[ n = \frac{f}{c} (t + x) \quad n = \frac{f'}{c} (t' + x') \]

- Using the Lorentz transformation and following the same logic we arrive at the shift in frequency for pulses traveling to the left.
  \[ f = f' \left( \frac{1 - v_{rel}}{1 + v_{rel}} \right)^{1/2} \]

- These two equations are equivalent. A sign accounts for whether the source is moving towards you or not (by convention positive velocities are those that are moving away)
Astronomers define the redshift, \( z \), of an object as:

\[
z = \frac{f_{\text{emit}} - f_{\text{obs}}}{f_{\text{obs}}}
\]

Note that

\[
1 + z = \frac{f_{\text{obs}}}{f_{\text{emit}}} = \frac{f_{\text{emit}} - f_{\text{obs}}}{f_{\text{obs}}} = \left( \frac{1 + v_{\text{rel}}}{1 - v_{\text{rel}}} \right)^{1/2}
\]

If the redshift is known we can solve for the velocity of the object. This gives

\[
v_{\text{rel}} = \frac{(1 + z)^2 - 1}{(1 + z)^2 + 1}
\]

Quasars have been measured to very high redshifts. For instance, a quasar measured to have \( z = 4.897 \), the above equation gives a velocity (relative to the speed of light), \( v_{\text{rel}} = 0.944 \).

At low velocities

\[
1 + z \approx \left( \frac{1 + v_{\text{rel}}}{1 - v_{\text{rel}}} \right)^{1/2} \approx \left( 1 + \frac{v_{\text{rel}}}{2} \right) \left( 1 + \frac{v_{\text{rel}}}{2} \right) = 1 + v_{\text{rel}} + \frac{1}{4} v_{\text{rel}}^2
\]

Ignoring terms greater than linear in \( v_{\text{rel}} \),

\[
z = v_{\text{rel}}
\]

Astronomers typically use the conventional velocity rather than one relative to the speed of light. This then becomes

\[
z = \frac{v_{\text{emit}}}{c}
\]

For nearby objects (galaxies) this approximation works well.
Early Astronomical History

- Slipher (~1912) noticed that spiral nebulae showed almost predominantly redshifts.
  - By 1925 he had radial velocities for 40 galaxies
- Hubble used the 100-inch telescope on Mt. Wilson to measure distances to 18 galaxies
  - Found linear relation between increasing redshift and increasing distance, now known as Hubble’s law

\[ H_o d = v_{\text{conv}} \sim cz \]

- Hubble found that the Universe is expanding!
  - The greater the distance, the higher the recession velocity
- A major goal of HST was to measure \( H_o \) accurately.
  - Currently, \( H_o \sim 70 \text{ km/s/Mpc} \)
  - Often used is \( H_o = 100 \ h \text{ km/s/Mpc} = (10^{10} \text{ yrs})^{-1} h \)