Spacetime and Inertial Reference Frames

Relativity and Astrophysics
Lecture 02
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Outline
- Events and Intervals
- Definitions
  - Proper time, Timelike, and Spacelike
- Sample Problems
- Inertial (Free Float) Reference Frames
  - Definition
  - Note – only applies of limited regions of time and space
- Test Particles
- Synchronizing clocks
- The “Observer”
- Some worked problems from end of chapters 1 and 2 of Spacetime Physics
- Readings
  - Spacetime Physics: Chapters 1 and 2
Events and Intervals – alone

- **Event**
  - Fundamental concept which represents something happening at a point in spacetime
  - Examples: two particles colliding, emission of a photon from an atom, lightning striking the engine of an airplane, etc.

- **Spacetime interval**
  - Observers, regardless of their velocity, will agree on the spacetime interval between two events!
  - If we travel at constant velocity from event to event then the spacetime interval is just the time separation (since the events all occur at the same location in our frame, the space separation is zero).

- **Intervals between events are all that is needed to specify the location of every event in spacetime.**

- **Using a reference frame is okay**
  - But the coordinates we specify are frame dependent

Proper Time, Timelike, & Spacelike

- **Proper Time**
  - Measured with a special clock (called the proper clock) that is carried so that it is present at each event as it occurs
  - Implies space separation is zero
  - Note – called wristwatch time (proper time) and wristwatch (proper clock) by textbook.

- **Timelike intervals**
  - A frame can be chosen so that the two events occur at the same location (time separation predominates over space separation).
  - Measured by using a wristwatch carried from one event to another in a special frame so that the event occur at the same place.

- **Spacelike intervals**
  - Spatial distance is always larger than temporal distance
  - Cannot choose a frame to make events occur at the same location
  - Measured by using a ruler to directly measure the distance between two events at the same time in your frame.
  - It is convention to change the sign of the interval when the spatial distance is larger than the temporal distance
  - \( \Rightarrow \) squared interval is never negative.

More later …
Lightspeed!

- The speed of light is defined (since 1983) to be:
  \[ c = 299,792,458 \text{ m/sec} \]
- This is very, very fast indeed!
- Some sample light travel times are given below.

<table>
<thead>
<tr>
<th>Route</th>
<th>Time</th>
<th>Time (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JFK – SFO</td>
<td>0.0139 sec</td>
<td>4,166,000</td>
</tr>
<tr>
<td>Around the Earth</td>
<td>0.1337 sec</td>
<td>40,080,000</td>
</tr>
<tr>
<td>Earth – Moon</td>
<td>1.281 sec</td>
<td>384,000,000</td>
</tr>
<tr>
<td>Sun – Earth</td>
<td>8.317 min</td>
<td>149,600,000,000</td>
</tr>
<tr>
<td>Sun – Proxima Centauri</td>
<td>4.280 yr</td>
<td>4.05x10^{16}</td>
</tr>
<tr>
<td>Sun – Andromeda Galaxy</td>
<td>2,280,000 yr</td>
<td>2.16x10^{22}</td>
</tr>
</tbody>
</table>

Note: A lightyear (lyr) is the distance light travels in 1 year = 9.46x10^{15} meter. Astronomers typically use parsecs (pc) where 1 pc = 3.26 lyr. Space is big!

Sample Problem: 1-2a

- A proton is moving at 0.75c with respect to your laboratory and passes through two detectors 2 meters apart. Events 1 and 2 are the transits through two detectors.
  1. What are the laboratory space and time separations between the two events, in meters?
  2. What are the space and time separations in the proton frame?

**Part 1:** The space separation in the lab is just 2 m, as given. The travel time (in meters) is

\[ t_{\text{lab}} = \frac{2 \text{ m}}{0.75} = 2.6667 \text{ m} \]

\[ t_{\text{lab}} \text{ (in meters)} = \frac{d_{\text{lab}}}{v_{\text{proton}}} \]

\[ v_{\text{proton}} = \frac{d_{\text{lab}}}{c (v_{\text{proton}} / c)} = \frac{d_{\text{lab}}}{0.75} \]

**Part 2:** Both events occur at the position of the proton

\[ \Rightarrow \text{space separation is 0 m in proton frame.} \]

We use invariance of spacetime interval to get time separation in the proton frame:

\[ t_{\text{proton}}^2 - d_{\text{proton}} = t_{\text{lab}}^2 - d_{\text{lab}} \]

\[ \Rightarrow t_{\text{proton}}^2 - (0 \text{ m}^2) = (2.6667 \text{ m})^2 - (2.0 \text{ m})^2 \]

\[ = (7.1111 - 4) \text{ m}^2 = 3.1111 \text{ m}^2 \]

\[ \Rightarrow t_{\text{proton}} = 1.764 \text{ m} \]
Sample Problem: 1-2b
A speeding rock from space streaks through Earth’s outer atmosphere, creating a fiery trail (Event 1) and crashes into the Sun (Event 2) 10 minutes later as observed in the Earth frame. Take the Sun-Earth distance to be $1.4960 \times 10^{11}$ meters.

1. In the Earth frame what are the space and time separation between the two events in minutes?
2. What are the space and time separations in the frame of the rock?

**Part 1:** The time separation is 10 minutes, as stated in the problem. The space separation (in minutes is):

\[ d_{\text{rock}} = \frac{d_{\text{sun-earth}}(\text{m})}{c(\text{m/sec})} \times \frac{1 \text{ min}}{60 \text{ sec}} = \frac{1.4960 \times 10^{11}}{29792458 \times 60} \approx 8.3169 \text{ light-minutes} \]

**Part 2:** Both events occur at the position of the rock

=> space separation is 0 m in rock frame.

Again, we use invariance of spacetime interval to get time separation in the proton frame:

\[ t_{\text{rock}}^2 = d_{\text{rock}}^2 - d_{\text{sun-earth}}^2 \]

\[ t_{\text{rock}}^2 = (10 \text{ min})^2 - (8.3169 \text{ min})^2 \]

\[ t_{\text{rock}}^2 = (100 - 69.1708) \text{ min}^2 = 30.8292 \text{ min}^2 \]

\[ t_{\text{rock}} = 5.5524 \text{ m} \]

This is the famous “twin paradox” – time goes by more slowly on the starship than the Earth.

Sample Problem: 1-2c
A starship leaves Earth (Event 1) and travel at 95% light speed, later arriving at Proxima Centauri (Event 2), which lies 4.3 light-years from Earth.

1. What are the space and time separation between the two events as measured in the Earth frame, in years?
2. What are the space and time separations in the frame of the starship?

**Part 1:** The distance separation is 4.3 light-years, as stated in the problem. The time separation is:

\[ t_{\text{earth}} = \frac{4.3 \text{ yr}}{0.95 (\text{lyr/yr})} = 4.53 \text{ years} \]

**Part 2:** Both events occur at the position of the starship

=> space separation is 0 m in starship frame.

Once more, we use invariance of spacetime interval to get time separation in the proton frame:

\[ t_{\text{ship}}^2 = t_{\text{earth}}^2 - d_{\text{sun-earth}}^2 \]

\[ t_{\text{ship}}^2 = (4.53 \text{ yr})^2 - (4.3 \text{ yr})^2 \]

\[ t_{\text{ship}}^2 = (20.52 - 18.49) \text{ yr}^2 = 2.03 \text{ yr}^2 \]

\[ t_{\text{ship}} = 1.42 \text{ years} \]

This is the famous “twin paradox” – time goes by more slowly on the starship than Earth.
Unity of Spacetime

- Space and time go together to form spacetime
  - Space is different for different observers
  - Time is different for different observers
  - Spacetime is the same for everyone
- Some physical quantities are the same for every observer
  - Spacetime, electric charge and particle mass
- Other physical quantities are relative and depend on the relative motion of the observers
  - Velocity, momentum, energy, separation in time, and separation in space
- Note – time and space are NOT identical in quality
  - One equal footing, but not the same. There is a minus sign in the spacetime interval!
- Our goal now is to develop our intuition about spacetime
  - We don’t deal with the effects of spacetime in everyday life.

Early Misconceptions about Space

- You can “shoot” people into space
  - Even if the space capsule didn’t melt, the acceleration would kill everyone inside.
- Rockets wouldn’t work
  - Need something to “push against”
  - Rockets work with just “Newtonian mechanics” – conservation of momentum
- Human can’t survive in space
  - Partly true – vacuum is a problem, and some “long problems” associated with zero-g
- You would feel gravity when orbiting the Earth
  - Essentially Jules Verne interpretation in “A Trip Around the Moon” – had some parts right and others not.
  - A piece of space junk will follow alongside a non-accelerating capsule but he had humans feeling gravity inside!
Free Fall (or free-float)

- If you were in “empty” space you would not experience the gravity of Earth.
  - You would be “floating” in space.
  - A tossed ball travels in a straight line (red line at right).
- Here on Earth we are always under the influence of gravity.
  - Toss a ball and it follows a parabolic arc (dotted line at right).
  - We see this arc because we are not under going our “natural” motion here on Earth.
  - The floor stops us from “falling.”
- Allow the room to “fall” with the ball and we see the ball traveling in a straight line.
- Empty space and free fall are the same
  - Weightlessness and free fall
  - A pencil “falls” when you let it go because you are not falling with it! If you in free fall the pencil would stay at rest with you without applying a force (your hand).

Orbits are in “Free Fall”

- When a spaceship orbits Earth astronauts feel weightless because they are in free fall.
  - In other words they experience the same feeling you would in a falling elevator. Why?
- In orbiting Earth, a spaceship is constantly falling towards Earth but it is also moving along the orbit (sideways).
  - Thus Earth’s surface drops away – at the same rate as the ship is falling! Result it doesn’t get any closer.
Microgravity Availability

- Link on candle flames in zero g: chemistry.about.com/od/chemistryfaqs/f/fire0gravity.htm

Drop Towers

- NASA GRC 2.2 second Drop Tower
  - 2.2 seconds
  - 24.1 m
  - 10^{-4} g

- NASA MSFC Drop Tube
  - 4.6 seconds
  - 105 m
  - 10^{-5} g

- ZARM Drop Tower
  - 4.74 seconds
  - 123 m
  - 10^{-5} g

- NASA GRC Zero Gravity Research Facility
  - 5.18 seconds
  - 145 m
  - 10^{-5} g

- Japan Microgravity Center (decommissioned)
  - 10 seconds
  - 490 m
  - 10^{-5} g

- After a bit dated 2001 presentation by Richard DeLombard of NASA Glenn Research Center (at Lewis Field, near Cleveland, Ohio).
JAMIC was established in March 1989 and began full operation in Kamisunagawa, Hokkaido in 1991. It handled roughly two drops per day. The rocket-shaped capsule is 7.85 m long, 1.8 m diameter and 5.5 ton in weight. It drops toward the bottom of 710 m drop shaft. Photos from: [http://www.gel.civil.nagasaki-u.ac.jp/text/example/ex34/ex34.html](http://www.gel.civil.nagasaki-u.ac.jp/text/example/ex34/ex34.html)

- Facility closed in ~ 2000 (attempts to keep it open failed), replaced with 50 m drop tower.